# Green function for a spin-1/2 particle in an external plane wave electromagnetic field



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**COMMENT**

# Green function for a spin-½ particle in an external plane wave electromagnetic field

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**Abstract.** The Green function for a spin-½ charged particle in the presence of an external plane wave electromagnetic field is calculated by algebraic techniques in terms of the free-particle Green function.

The Green function for a charged particle in the presence of an external plane-wave electromagnetic field was calculated by Schwinger [1]. In a recent paper [2] it was shown that the spin-0 Green function can be obtained from a different viewpoint. In place of formulating the problem using non-commuting operators $\Pi_\mu$ it is possible to formulate it using commuting operators $\hat{\Pi}_\mu$ which are related to the free-particle operators $p_\mu$ by a unitary transformation. The explicit calculation of the unitary transformation led to the final result giving the Green function in the interacting case in terms of the free-particle one. In the present comment we extend our results to the spin-½ case.

The external plane-wave field is taken to be

$$F_{\mu\nu}(x)=f_{\mu\nu}\frac{\mathrm{d}A}{\mathrm{d}\xi}=f_{\mu\nu}F(\xi) \tag{1}$$

where

$$\xi=n\cdot x \qquad n^2=0. \tag{2}$$

Also

$$n^\mu f_{\mu\nu}=n^{\mu *}f_{\mu\nu}=0 \tag{3}$$

so that

$${}^*f_{\mu\lambda}f^\lambda_{\ \nu}=0 \tag{4}$$

and we fix the normalization of $f_{\mu\nu}$ by

$$f_{\mu\lambda}f^\lambda_{\ \nu}={}^*f_{\mu\lambda}{}^*f^\lambda_{\ \nu}=n_\mu n_\nu. \tag{5}$$

The vector potential $A_\mu(x)$ is chosen to be

$$A_\mu(x)=f_{\mu\nu}(x-x')^\nu\chi(\xi,\xi') \tag{6}$$

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where

$$\chi(\xi, \xi') = \frac{A(\xi)}{\xi - \xi'} - \frac{1}{(\xi - \xi')^2} \int_{\xi'}^{\xi} A(\eta)\, d\eta \tag{7}$$

and $x'$ is a convenient reference point.

The Green function for a spin-$\frac{1}{2}$ charged particle satisfies

$$(\not{\Pi} - m)s(x, x') = (i\not{\partial} - e\not{A} - m)s(x, x^1) = \delta^4(x - x^1). \tag{8}$$

Alternatively if we put

$$S(x, x') = (\not{\Pi} + m)S_1(x, x') \tag{9}$$

then we have to solve

$$\left(\Pi^2 - m^2 + \frac{e}{2}\sigma \cdot F\right)S_1(x, x') = \delta^4(x - x') \tag{10}$$

where

$$\sigma \cdot F = \sigma_{\mu\nu}F^{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]F^{\mu\nu}. \tag{11}$$

Putting

$$S_1(x, x') = \int_{-\infty}^{+\infty} \exp(-im^2 s)\, \Sigma_s(x, x')\, ds \tag{12}$$

one can replace (10) by

$$(i\partial_s + \Pi^2 + \sigma \cdot F)\Sigma_s(x, x') = \delta(s)\delta^4(x - x'). \tag{13}$$

In our earlier paper [2] we proved results appearing in the equations (14)-(20) which follow. The 'momenta' $\hat{\Pi}_\mu$ defined by

$$\hat{\Pi}_\mu = \Pi_\mu + \frac{ef_{\mu\nu}\Pi^\nu A(\xi)}{n \cdot \Pi} + \frac{e^2 A^2(\xi) n_\mu}{2n \cdot \Pi} \tag{14}$$

satisfy

$$[\hat{\Pi}^\mu, \hat{\Pi}^\nu] = 0 \tag{15}$$

and

$$\hat{\Pi}^2 = \Pi^2. \tag{16}$$

Also

$$\hat{\Pi}_\mu = Up_\mu U^{-1} \tag{17}$$

where

$$U = \exp\left\{-\frac{ie}{n \cdot p}\Gamma(\xi, \xi')A \cdot p - \frac{ie^2}{2n \cdot p}\Omega(\xi, \xi')\right\} \tag{18}$$

with

$$\Gamma\chi = A - (\xi - \xi')\chi \tag{19}$$

and

$$\Omega(\xi, \xi') = \int_{\xi'}^{\xi} A^2(\eta)\,\mathrm{d}\eta - \frac{1}{\xi - \xi'}\left(\int_{\xi'}^{\xi} A(\eta)\,\mathrm{d}\eta\right)^2. \tag{20}$$

Now if we put

$$\tilde{\Pi}^\mu = \hat{\Pi}^\mu + \frac{e}{4}\frac{\sigma \cdot F}{n \cdot \Pi} n^\mu \tag{21}$$

then

$$[\tilde{\Pi}^\mu, \tilde{\Pi}^\nu] = 0 \tag{22}$$

and

$$\tilde{\Pi}^2 = \Pi^2 + \frac{e}{2}\sigma \cdot F. \tag{23}$$

One can show that

$$\tilde{\Pi}^\mu = V\hat{\Pi}^\mu V^{-1} \tag{24}$$

where

$$V = \exp\left\{\mathrm{i}e\frac{A(\xi)\sigma \cdot F}{4np}\right\} \tag{25}$$

so that

$$\tilde{\Pi}^\mu = Wp^\mu W^{-1} \tag{26}$$

where

$$W = UV. \tag{27}$$

We note that if

$$\Pi^\mu = \Lambda^\mu_\nu \hat{\Pi}^\nu \tag{28}$$

then $\Lambda^\mu_\nu$ formally behaves like a Lorentz transformation. Also

$$V^{-1}\gamma^\mu V = \Lambda^\mu_\nu \gamma^\nu \tag{29}$$

and

$$V^\dagger = \gamma^0 V^{-1} \gamma^0 \tag{30}$$

so that $V$ is the SL(2, C) transformation which corresponds to the Lorentz transformation $\Lambda$. Of course both $\Lambda$ and $V$ are now field dependent.

We may use the above results to solve equation (13), immediately getting

$$S_1(x, x') = W(x)S_{10}(x, x')W^{-1}(x') \tag{31}$$

where $S_{10}$ corresponds to the free-particle case. It may be written explicitly by using

$$\Sigma_{s_0}(x, x') = \frac{\mathrm{i}\theta(s)}{(4\pi)^2 s^2}\exp\left(-\mathrm{i}\frac{(x-x')^2}{4s}\right). \tag{32}$$

It is easy to see that one gets $S$ in the form obtained by Schwinger.

One can define a 'position' operator

$$\tilde{x}_\mu = W x_\mu W^{-1} \tag{33}$$

which is conjugate to $\tilde{\pi}_\mu$. Defining

$$\tilde{\gamma}^\mu = V \gamma^\mu V^{-1} \tag{34}$$

we have that $\tilde{\Pi}_\mu$ and

$$\tilde{M}_{\mu\nu} = \tilde{x}_\mu \tilde{\Pi}_\nu - \tilde{x}_\nu \tilde{\Pi}_\mu + \frac{i}{2}[\tilde{\gamma}^\mu, \tilde{\gamma}^\nu] \tag{35}$$

generate a representation of the Poincaré group.

The use of second-order formalism is not necessary. One can easily verify that

$$\not{\Pi} = W \not{p} W^{-1} \tag{36}$$

so that equation (8) may be solved directly. It is, however, difficult to see how one can obtain equation (36) without starting from the second-order equation.

To conclude, we have shown that the technique used in our earlier paper [2] can be generalized to the spin-$\frac{1}{2}$ case. It should be noted, however, that the transformation $W$, in contrast to the earlier case, is a non-unitary Bogoliubov transformation.

It is interesting to observe that equation (36) and a similar one in the spin-0 case allow us to apply Fukikawa's technique and the path integral formalism to calculate the generating functional of Green functions exactly. This will be reported elsewhere.

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## References

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